Complex Numbers Cribsheet

Cartesian Form

Basic form of a complex number is

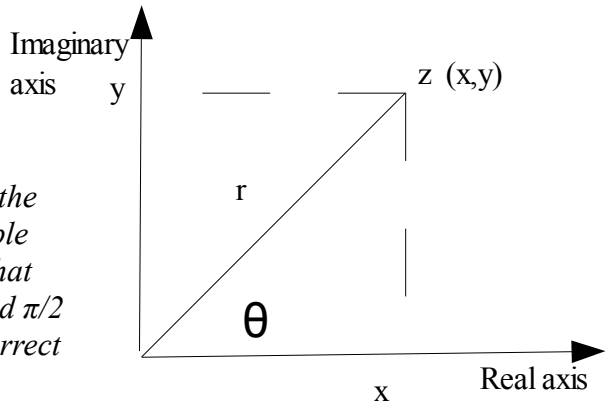
$$z = x + yi$$

$$\text{where } i = \sqrt{(-1)}$$

where x is the real component and y the imaginary component.
The modulus (or radius vector), and the argument are

$$r = \sqrt{(x^2 + y^2)}$$

$$\arg z = \arctan(y/x)$$



respectively. Whilst z can be in any of the 4 quadrants, by convention the principle range of θ is between -π and π. Note that arctan is only defined between -π/2 and π/2 Care must be taken to calculate the correct value of arg z.

Complex numbers can be added, differenced, multiplied and divided using the normal rules of algebra, remembering that $i^2 = -1$.

Polar Form

$z = r(\cos\theta + i \sin\theta)$ (or using Euler's equation $z = r e^{i\theta}$)

modulus = r argument = θ

In polar form, the product and quotient of complex numbers is simplified

if $w = s(\cos\varphi + i \sin\varphi)$

$zw = rs(\cos(\theta+\varphi) + i \sin(\theta+\varphi))$
$z/w = r/s (\cos(\theta-\varphi) + i \sin(\theta-\varphi))$

Complex Conjugate

The complex conjugate to $z = x + iy$ is $z^* = x - iy$

to $z = r(\cos\theta + i \sin\theta)$ is $z^* = r(\cos\theta - i \sin\theta)$

note that zz^* is always real and $= x^2 + y^2 = r^2$

Complex roots of equations with real coefficients always occur in conjugate pairs (eg a+ib, a-ib)

Euler's equation

$e^{i\theta} = \cos\theta + i \sin\theta$

De Moivre's Theorem

If $z = r(\cos\theta + i \sin\theta)$ *then* $z^n = r^n(\cos n\theta + i \sin n\theta)$

De Moivre's theorem applies for all rational values of n.